

# Dependent indefinites – the view from sign language

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Paper: [www.semanticsarchive.net/Archive/jYxMzJkN/](http://www.semanticsarchive.net/Archive/jYxMzJkN/)

## 1 Overview

- In natural language, **indefinites** introduce individuals into the discourse context.
  - (1) I read a book. (It was interesting.)
  - (2) I read three books. (They were interesting.)
- In many languages, indefinites may be inflected to create a **dependent indefinite**.
  - Indicates that the value of the DP varies w.r.t. something else in the sentence or context.
- (3) **Telugu** (Balusu 2006)

pilla-lu renDu-renDu kootu-lu-ni cuus-ee-ru.  
kids two-two monkey-Pl-Acc see-Past-3PPL  
'(The) kids saw two monkeys each.'
- (4) **ASL**

BOYS IX-arc-a READ ONE-arc-a BOOK.  
'The boys read one book *each*.'
- *Distributive meaning*: two monkeys per kid; one book per boy

### Questions for today

1. What is the relation between a dependent term and its licensor? (Anaphoric or indirect?)
2. Does the distributive meaning reside in the dependent indefinite itself, or is it parasitic on a (possibly covert) distributivity operator elsewhere in the sentence?
3. (How does a dependent indefinite 'see outside' the scope of a distributive operator?)

## The point of view from sign language

- Dependent indefinites in ASL fit into a broader typology:
  - interpretation
  - licensing conditions
  - even morphological marking via reduplication
- Additionally, ASL pattern employs the use of space.
  1. Overt representation of the relation between a dependent indefinite and its licensor.
    - \* Anaphoric!
  2. Spatial marking of dependency also appears on the adjectives SAME and DIFFERENT.
    - \* Intrinsically distributive!
- Consequences for recent theories of dependent indefinites.

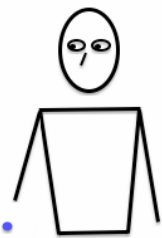
## 2 Visible dependency

- The meaning of dependent indefinites can be characterized by a **variation condition**:
  - the value taken by the dependent indefinite varies with the atoms of the plural licensor.
- The variation condition can be seen in the fact that collective readings become unavailable with dependent indefinites.

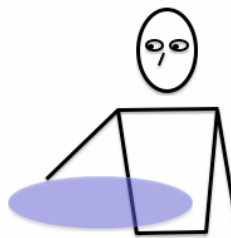
- (5) a. ALL BOY LIFT ONE TABLE. ✓collective ✓distributive  
b. ALL BOY LIFT ONE-arc TABLE. \* collective ✓distributive  
‘All the boys lifted a table.’

### The use of space in ASL

- Singular individuals indexed at points in space. (Lillo-Martin and Klima 1990, *i.a.*)
- Plurals are indexed over areas of space.



singular locus



plural locus

## A spatial representation of dependency:

- Dependent indefinites are obligatorily signed over the same area of space as their licensor.

(6) EACH-a PROFESSOR SAID ONE-arc-a STUDENT WILL RECEIVE B.

‘Each professor said that one student will receive a B.’

(7) ?? EACH-a PROFESSOR SAID ONE-arc-b STUDENT WILL RECEIVE B.

‘Each professor said that one student *from each salient group* will receive a B.’

## Removing ambiguity in ASL

- Since the arc-motion agrees with licensor, can specify **what the indefinite is dependent on**.
- Consider a sentence with two potential licensors.

(8) **Hungarian** (p.c. Dániel Szeredi; four speakers)

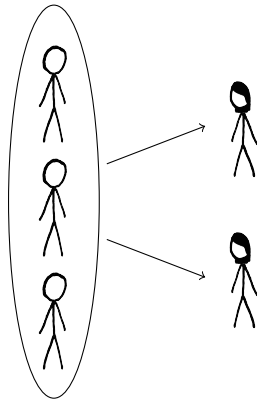
A fiúk két-két könyvet adtak a lányoknak.

The boys two-two book give.3Pl the girls

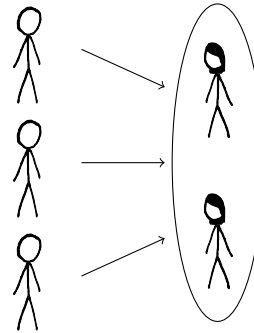
‘The boys gave the girls two books each.’

- Judgement: ‘two-two’ can depend on either boys or girls.

a. Distribution across the girls.  
‘To Mary, from the boys’



b. Distribution across the boys.  
‘To the girls, from John.’



- With the use of space, ASL can disambiguate!

(9) ALL-a BOY-a GAVE ALL-b GIRL-b ONE-arc-b BOOK.

‘All the boys gave all the girls one book *per girl*.’

- Reading (b), with distribution across the boys, is *not possible*.

## Theoretical import

- **Debate in the literature:** what is the relation between a dependent term and its licensor?
- Two sides:

<b>Anaphoric link</b> (like pronouns)	<b>Indirect relation</b> (like NPIs)
Brasoveanu & Farkas 2011	Balusu 2006
Brasoveanu 2011 ( <i>different</i> )	Henderson 2014
Barker 2007 ( <i>same</i> )	Cable 2014

- **New conclusion:** the ASL data is overtly anaphoric.

## 3 A compositional puzzle: whence the distributive force?

### A licensing puzzle

- A key property: **licensing**
- In many languages, dependent indefinites are...
  - ...licensed by plurals,
  - ...licensed by distributive operators,
  - ...ungrammatical when all arguments are singular.
- This holds in some form in...
  - Kaqchikel (Henderson 2014), Hungarian (Farkas 1997), Romanian (Brasoveanu & Farkas 2011), Albanian (Rushiti 2015), Telugu (Balusu 2006), some dialects of English (Champollion 2015a), and ASL (this work).

### Licensing examples

#### (10) **Kaqchikel Mayan** (Henderson 2014)

- Xeqatij ox-ox wäy.  
we-eat three-three tortilla  
'We each ate three tortillas.'
- Chikijujunal ri tijoxela' xkiq'etej ju-jun tz'i'.  
each the students hugged one-one dog  
'Each of the students hugged a dog.'
- \* Xe'inchäp ox-ox wäy.  
I-handle three-three tortilla  
*Desired reading:* 'I took (groups of) three tortillas.'

(11) **Telugu** (Balusu 2006)

- a. Pilla-lu renDu-renDu kootuluni cuuseeru  
kid-Pl two-two monkeys saw  
'(The) kids saw two monkeys each.'  
*Two readings: 'participant key' and 'temporal key.'*
- b. Prati pillavaaDu renDu-renDu kootuluni cuuseeDu  
Every kid two-two monkeys saw  
'Every kid saw two monkeys (each).'  
*Two readings: 'participant key' and 'temporal key.'*
- c. Raamu renDu-renDu kootuluni cuuseeDu  
Ram two-two monkeys saw  
'Ram saw two monkeys each.'  
*Only 'temporal key' reading.*

(12) **Albanian** (Rushiti 2015)

- a. Fëmijët kanë parë nga pesë mace.  
children-the have seen DIST five cats  
'The children have seen five cats each'
- b. Në çdo dhomë kishte nga dy fotografi.  
in every room there-were DIST two photos  
'There were two (different) photos in each room'
- c. \* Në dhomë kishte nga dy fotografi.  
in room there-were DIST two photos  
*Desired reading: 'There were two (different) photos in the room.'*

(13) **English**

- a. The boys saw two zebras each.
- b. % Every job candidate was in the room for fifteen minutes each.
- c. \* Ariella saw two zebras each.

**American Sign Language fits in perfectly:**

(14) **ASL**

- a. BOYS IX-arc-a READ ONE-arc-a BOOK.  
'The boys read one book each.'
- b. EACH-EACH-a PROFESSOR NOMINATE ONE-redup-a STUDENT.  
'Each professor nominated one student.'
- c. \* JOHN-a READ ONE-arc-a BOOK.  
*Desired reading: 'John read one book.'*

## A compositional puzzle

- Quantifiers like English *every* distribute down to atomic parts.

### (15) English

- a. The boys gathered.
- b. \* Every boy gathered.
- c. \* Edith gathered.

### (16) ASL

- a. MY FRIENDS, IX-arc-a GATHER.  
'My friends gathered.'
- b. \* EACH STUDENT MY CLASS GATHER.  
'Each student in my class gathered.'
- c. \* JOHN GATHER.  
'John gathered.'

## A compositional puzzle

- Dependent indefinites under distributive operators seem to be puzzlingly redundant (e.g. Balusu 2006, Oh 2005).
  - With a plural licensor, they seem to contribute distributive force themselves.
  - Under distributive operators, they appear to be semantically vacuous.
- If there are cases in which they are semantically vacuous, then why can't they appear innocently under singular subjects?

## Two possible directions

**Option 1:** Treat licensing by distributives as the 'base case.'

(Brasoveanu and Farkas 2011, Henderson 2014)

- The at-issue meaning of a dependent indefinite is equivalent to that of a plain indefinite.
- A syntactic or semantic constraint (e.g. 'distributive concord') requires the indefinite to scope under a distributive operator.
- Licensing by plurals arises via a covert distributivity operator.

(17) The boys DIST [saw two-two zebras].

**Option 2:** Treat licensing by plurals as the 'base case.'

(Balusu 2006, Cable 2014)

- The at-issue meaning of a dependent indefinite is itself quantificational/distributive.

(18)  $\llbracket$ two-two books $\rrbracket$  = Given a licensor  $X$ ,  
presupposing that  $X$  is nonatomic,  
 $\forall$  atomic parts  $x$  of  $X$ , there are two books associated with  $x$

- For distributive operators:
  - The dependent indefinite is able to ‘escape’ from the distributive scope, to get access to a higher plurality.
  - This plurality is made available by the compositional system.

### A problem for Option 1

(Option 1 = plural licensors require a covert distributivity operator)

- Distributive operators generally assumed to appear over VP.
- However, dependent indefinites may be conjoined with plain indefinites that are interpreted cumulatively.

(19) **Hungarian** (p.c. Dániel Szeredi)

A diákok két előételt és **egy-egy** főételt rendeltek.

The students two appetizers and one-one main-dish ordered.

‘The students ordered two appetizers in total, and as many main dishes as students’

(20) **Tamil (Chennai dialect)** (p.c. Anushree Sengupta)

Mānavarkkal thankalai kaga **oru-oru** appetizer o irenDu desserts share-panna order pannagu.

students themselves for one-one appetizer and two desserts share-do order did

‘The students ordered one appetizer each for themselves and two desserts to share.’

- If the dependent indefinite scopes under a covert distributive operator, the plain indefinite must do so, too, incorrectly entailing twice as many appetizers as students.

### Support for Option 2

(Option 2 = dependent indefinites are themselves distributive)

- **Observation:** English *same* shows the same distributional pattern as dependent indefinites

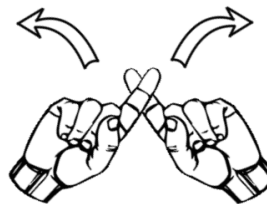
(21) English *same* (on internal reading):

- The students gave the same answer.
- Each student gave the same answer.
- \* Edith gave the same answer.

- **Observation:** in ASL, dependent indefinites, SAME, and DIFFERENT are morphologically unified.



SAME



DIFFERENT

- SAME/DIFFERENT show the same pattern of spatial agreement.

(22) BOY THEY-arc-a READ {ONE/SAME/DIFFERENT}-arc-a BOOK.  
'The boys read {one book each/the same book/different books}.'

- Like for dependent indefinites, ASL may remove ambiguity with multiple licensors.

(23) Every boy gave every girl the same book.

- a. *Reading 1*: unimaginative boys
- b. *Reading 2*: unlucky girls

(24) Every boy gave every girl a different book.

(Bumford and Barker 2013)

(25) BOYS IX-arc-a EACH GIVE-alt-b ALL-b GIRL-b SAME-arc-b BOOK.

- Only 'Reading 1': same with respect to the girls.

**Theoretical conclusion:** Dependent indefinites and the adjectives *same* and *different* should be treated in fundamentally the same way.

- Consider the meaning of *same*.

(26) Each student lifted the same table.

- A different table-lifting for each boy; in this sense, variation with respect to the licensor.
- But of course, this is not all; *same* compares the tables lifted by each boy.
  - For each pair of boys, they lifted the same table.
- This meaning is inherently quantificational.

**Thus...**

- *Same* must be given inherently quantificational meaning.
- Morphological parallels in ASL suggest that dependent indefinites should be treated analogously.



## The proposal in a nutshell

- Dependent indefinites introduce a plurality into a discourse.
- Two components of meaning:
  - **Presupposition:** the plurality can be divided into subpluralities that vary with respect to the atoms of a licenser.
  - **At-issue:** each of these subpluralities is of a given cardinality.
- Licensing by *each* is achieved by QR of the dependent indefinite, letting it scope outside the distributive operator.
  - Critically, the framework of **Plural Compositional DRT** allows the semantics to make reference to the functional dependency even outside of the distributive scope.

## 4 Introducing Plural Compositional DRT

### Background: dynamic semantics

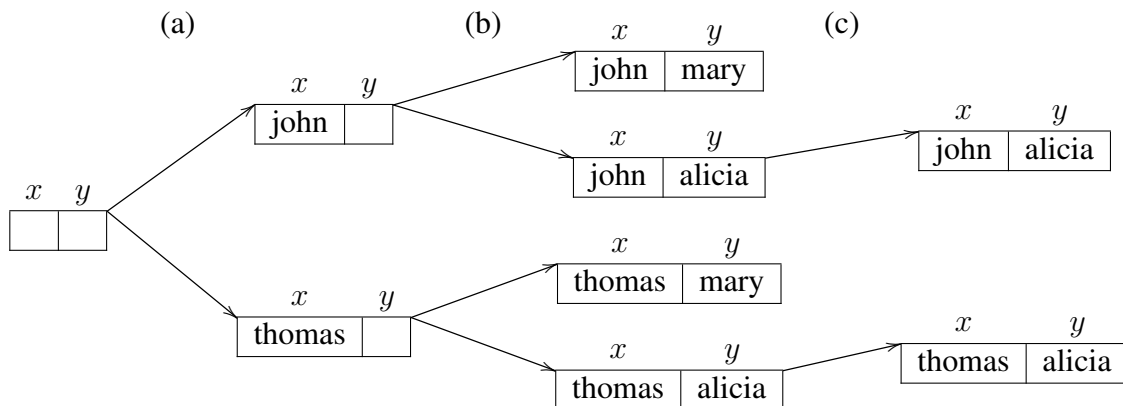
- **Dynamic semantics:** discourse referents represented as the values of an assignment function,  $g$  (essentially, a list).

$$- \quad g = \begin{array}{|c|c|c|} \hline x & y & z \\ \hline \text{john} & \text{mary} & \square \\ \hline \end{array} \dots \quad (\text{Groenendijk \& Stokhof 1991})$$

- Passed through discourse: the output context of one sentence is the input context of the next.
- Indefinites add new individuals to the list. Pronouns retrieve elements from the list

### Standard dynamic semantics, an example

(27) (a)  $A^x$  boy entered. (b)  $A^y$  girl exited. (c)  $She_y$  was angry.



## Universals in dynamic semantics

### Standard dynamic semantics:

- Universals *every* and *each* taken to be ‘externally static.’
  - Indefinites in their scope are not available to later discourse.
- At a first approximation, this seems to be correct:

(28) \* Every<sup>x</sup> farmer owns a<sup>y</sup> donkey. It<sub>y</sub> kicked me in the shin.

- But...

### Quantificational subordination (Heim 1990, Brasoveanu 2006)

(29) Two<sup>x</sup> farmers each own a<sup>y</sup> donkey.  
Neither of them<sub>x</sub> treat it<sub>y</sub> very well.

- The pronoun *it* is anaphoric to the indefinite *a donkey*, yet it doesn’t refer to a particular donkey or to the set of all donkeys.
  - It picks out the same correspondence that was introduced by the first sentence.

### Dynamic Plural Logic; Plural Compositional DRT

(van den Berg 1996, Nouwen 2003, Brasoveanu 2006)

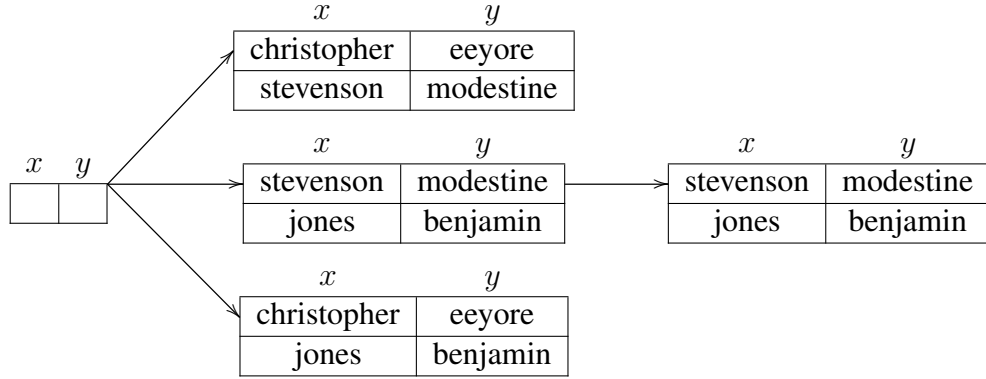
- We need to be able to ‘re-open’ the scope of a universal.
- Instead of just *checking* that there is one donkey per farmer, the system must *store* this representation.
- Instead of passing assignment functions through the discourse, it passes *sets* of assignment functions.

–  $G =$

$x$	$y$
christopher	eeyore
jones	benjamin

 ...

- $G, H$  are variables over these ‘information states’ (i.e. tables).
- (30) Two<sup>x</sup> farmers each own a<sup>y</sup> donkey.  
Neither of them<sub>x</sub> treat it<sub>y</sub> very well.



### System summary

- I adopt the Plural Compositional DRT of Brasoveanu 2006; full definitions in Appendix A.

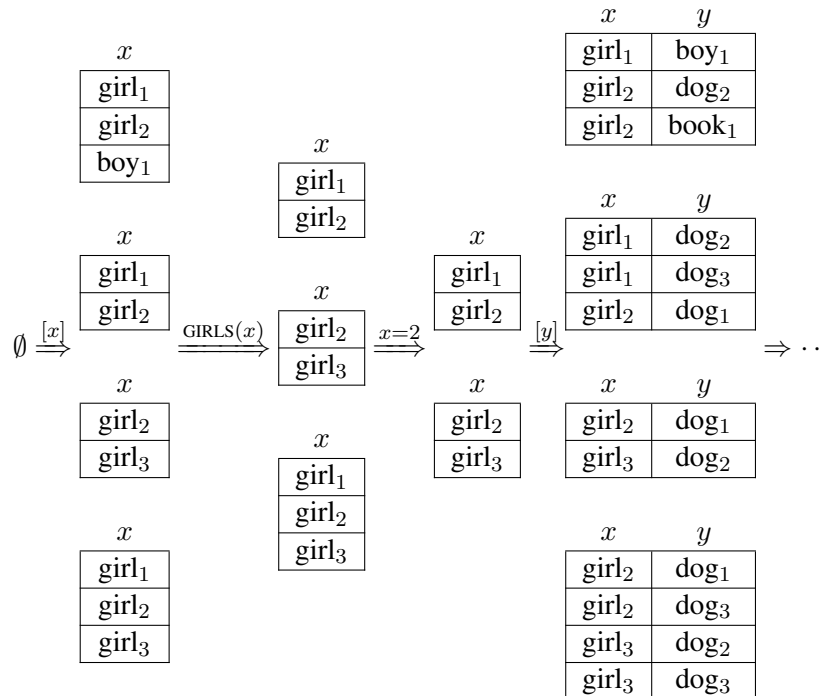
*Informally, ...*

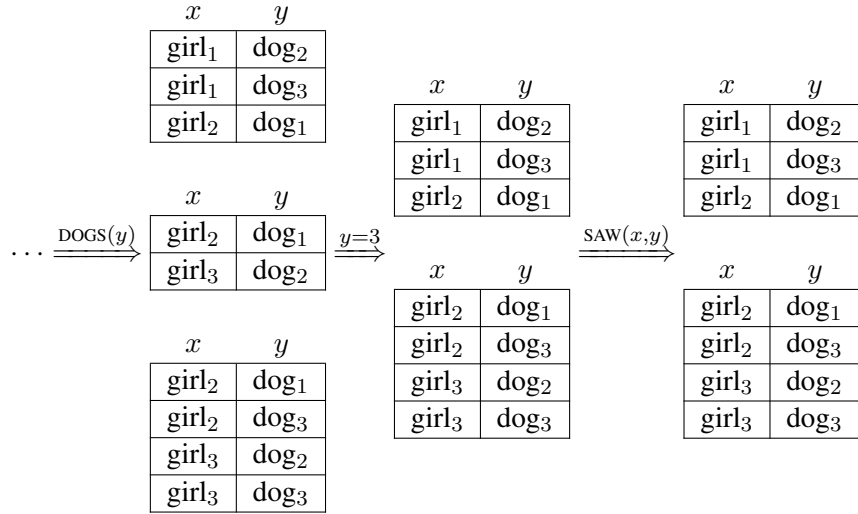
- $[x]$  introduces individuals across  $G$  at index  $x$ .
- Predicates test that a certain property holds for the values in each  $g \in G$ .
- Numerals are tests of the cardinality of the set of distinct values of some index  $x$  in  $G$ .

### An example

(31) a. Two <sup>$x$</sup>  girls saw three <sup>$y$</sup>  dogs.

b.  $[x] \wedge \text{GIRLS}(x) \wedge x = 2 \wedge [y] \wedge \text{DOGS}(y) \wedge y = 3 \wedge \text{SAW}(x, y)$

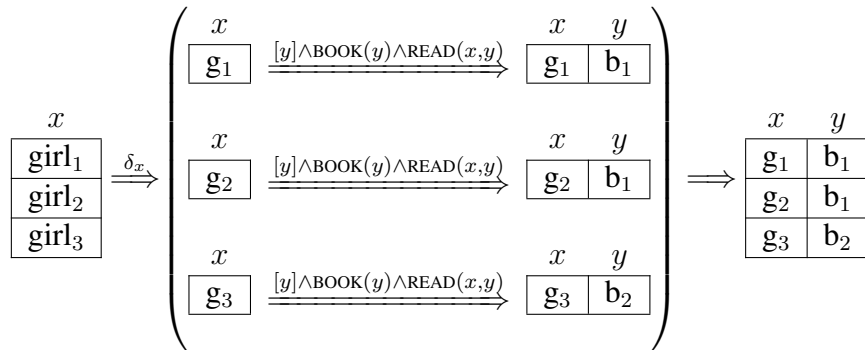




### The distributive operator

- The distributive operator  $\delta_x(\varphi)$  divides up a table with respect to the values of  $x$ , evaluates  $\varphi$  on each of these substates in parallel, then gathers up the resulting states.

- (32) a. ... each read a book.  
 b.  $\delta_x([y] \wedge \text{BOOK}(y) \wedge \text{READ}(y, x))$



### From earlier: The proposal in a nutshell

- Dependent indefinites introduce a plurality into a discourse.
- Two components of meaning:
  - **Presupposition:** the plurality can be divided into subpluralities that vary with respect to the atoms of a licenser.
  - **At-issue:** each of these subpluralities is of a given cardinality.
- We can now translate these statements into PCDRT.

### Substates based on the licenser

- First, an information state is divided up with respect to the values of the licenser.  $\{G|_{x=d}(y)\}$  is a set of sets.

$$(33) \quad G|_{x=d}(y) := \{g(y) | g \in G \ \& \ g(x) = d\}$$

- Below,  $x$  corresponds to the licenser;  $y$  corresponds to the dependent indefinite.

$$(34) \quad \text{a. } G = \begin{array}{cc} x & y \\ \hline a & e \\ \hline a & f \\ \hline b & g \\ \hline b & h \\ \hline c & i \\ \hline c & j \end{array} \quad \text{b. } \{G|_{x=d}(y)\} = \{\{e, f\}, \{g, h\}, \{i, j\}\}$$

a	e
a	f
b	g
b	h
c	i
c	j

### Presupposition of dependency

- **Presupposition:** the plurality can be divided into subpluralities that vary with respect to the atoms of a licenser.

$$(35) \quad \mathbf{outside}(y/x) > 1 \quad := \quad \lambda GH.G = H \quad \& \quad |\{G|_{x=d}(y)\}| > 1$$

- Equivalent to Nouwen's (2003) definition of dependency.

### Cardinality assertion

- **At-issue:** each of these subpluralities is of a given cardinality.

$$(36) \quad \mathbf{inside}(y/x) = n \quad := \quad \lambda GH.G = H \quad \& \quad \forall T \in \{G|_{x=d}(y)\}.|T| = n$$

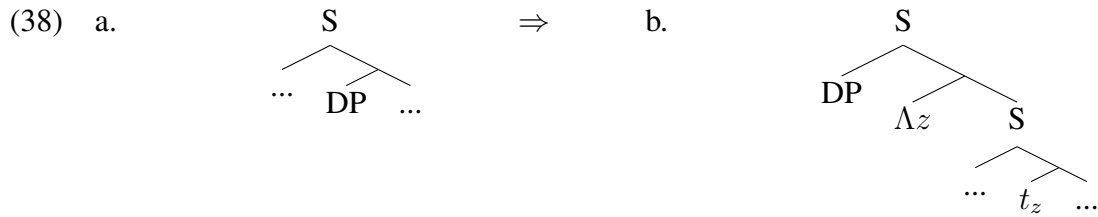
### Lexical definition of a dependent indefinite

$$(37) \quad \llbracket \text{two-two}_x^y \rrbracket = \lambda NP.[y] \wedge N(y) \wedge P(y) \wedge \mathbf{outside}(y/x) > 1 \wedge \mathbf{inside}(y/x) = 2$$

- Note that the two cardinality checkers are evaluated *after* the two predicates are introduced.
  - This allows the cardinality checkers to refer to an index that is introduced by an argument of the dependent indefinite.
  - This is the reflection in my analysis of Henderson's (2014) insight that the plurality condition of a dependent indefinite is somehow 'postsuppositional.'

## Quantifier raising

- I assume that quantifiers can move by Quantifier Raising (QR).



### Example 1

- (39) a. Three<sup>x</sup> students saw two-two<sup>y</sup> zebras.  
 b.  $[x] \wedge \text{STUDENTS}(x) \wedge [y] \wedge \text{ZEBRAS}(y) \wedge \text{SAW}(y)(x) \wedge$   
 $\text{inside}(x) = 3 \wedge \text{outside}(y/x) > 1 \wedge \text{inside}(y/x) = 2$

(40)

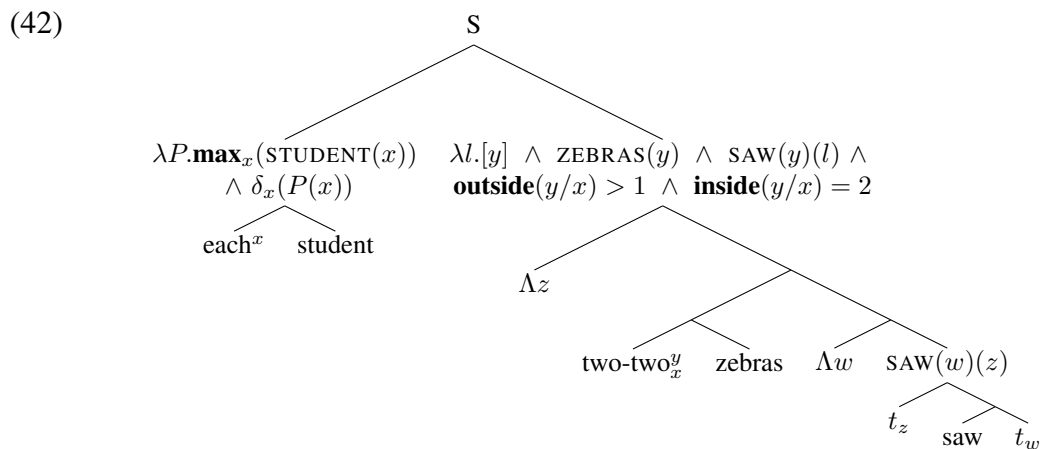
student <sub>1</sub>	zebra <sub>1</sub>
student <sub>1</sub>	zebra <sub>2</sub>
student <sub>2</sub>	zebra <sub>1</sub>
student <sub>2</sub>	zebra <sub>2</sub>
student <sub>3</sub>	zebra <sub>1</sub>
student <sub>3</sub>	zebra <sub>3</sub>

student <sub>1</sub>	zebra <sub>1</sub>
student <sub>1</sub>	zebra <sub>2</sub>
student <sub>2</sub>	zebra <sub>3</sub>
student <sub>2</sub>	zebra <sub>4</sub>
student <sub>3</sub>	zebra <sub>5</sub>
student <sub>3</sub>	zebra <sub>6</sub>

...

### Example 2 (unsuccessful derivation)

- (41) a. Each<sup>x</sup> student saw two-two<sup>y</sup> zebras.  
 b.  $\mathbf{\max}_x(\text{STUDENT}(x)) \wedge$   
 $\delta_x([y] \wedge \text{ZEBRAS}(y) \wedge \text{SAW}(y)(x) \wedge$   
 $\text{outside}(y/x) > 1 \wedge \text{inside}(y/x) = 2)$

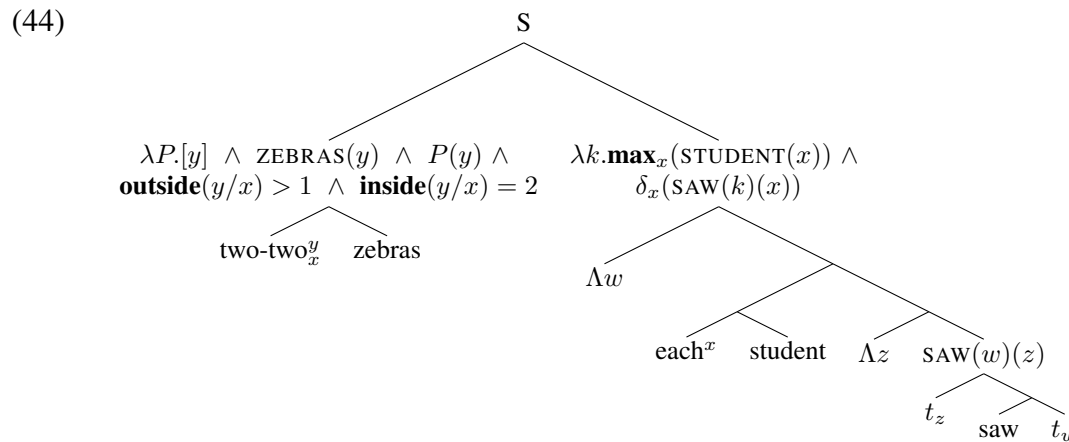


- Observe: the variation condition—i.e., the condition that **outside**( $y/x$ ) > 1—appears inside the distributive scope of  $\delta_x$ .
  - It is evaluated with respect to a substate of  $G$  where  $x$  is restricted to a single value.
  - The variation condition cannot be met, and the derivation fails.

### Example 2 (successful derivation)

- The dependent indefinite takes scope outside the distributive operator.

- (43) a. Each <sup>$x$</sup>  student saw two-two <sup>$y$</sup>  <sub>$x$</sub>  zebras.  
 b.  $[y] \wedge \text{ZEBRAS}(y) \wedge \mathbf{\max}_x(\text{STUDENT}(x)) \wedge$   
 $\delta_x(\text{SAW}(y)(x))$   
 $\wedge \mathbf{\text{outside}}(y/x) > 1 \wedge \mathbf{\text{inside}}(y/x) = 2$



- The variation condition ‘**outside**( $y/x$ ) > 1’ appears after distributive scope has closed, giving it access to the full set of values of  $x$  and  $y$ .

## 5 Theoretical payoff: how to take scope

- The essential insight for licensing by *each* comes from Henderson 2014.
  1. Dynamically tracking dependency relations with PCDRT.
  2. Evaluating the variation condition *after* the distributive scope has closed.
- However, on his analysis, dependent indefinites have the same at-issue content as plain indefinites. (They’re not distributive.)
- For Henderson 2014, result is a kind of ‘split-scope’:
  - At-issue content must scope below the distributive operator.
  - The variation condition must scope above it.

## Postsuppositions?

- Henderson: the variation condition is a **postsupposition**. (Brasoveanu 2012)
- Formally, postsuppositions are a special *kind* of meaning. (By analogy with presuppositions.)
  - Instead of being evaluated *in situ*, they are passed through the dynamic system until a later operator triggers their evaluation.

## On the other hand:

- The current analysis, with a distributive at-issue component, does not require separation of the two components of meaning; standard QR works, with no need for postsuppositions.
- *Further prediction of ‘standard scope-taking’*: sensitivity to **scope islands**
  - Dependent indefinites are licensed by distributive operators by scoping over them.
  - Thus: ungrammaticality when an island boundary (indicated below with ⟨·⟩) intervenes between a dependent indefinite and its potential licenser

**Hungarian** (p.c. Márta Abrusán, two speakers)

- (45) Minden professzor két-két diákról mondta, hogy meglepné ha **⟨diplomát szereznének⟩**.  
every professor two-two students-of said that surprised if diploma receive  
‘Every professor said of two students (each) that he would be surprised if they graduated.’
- (46) \* Minden professzor azt mondta, hogy meglepné, ha **⟨két-két diák diplomát szerezne⟩**.  
every professor DEM said that surprised if two-two student diploma receive  
‘Every professor said that he would be surprised if two students (each) graduated.’
- To my knowledge, not predicted by any other theory.

## 6 Summary

- I addressed the following architectural questions:
  1. Do dependent indefinites have an anaphoric component?
  2. Are dependent indefinites quantificational?
- The latter of these turns out to be connected to a third architectural question:
  3. Do dependent indefinites see outside of distributive operators via postsuppositions or standard scope?
- *My answers were:*

Dependent indefinites have an anaphoric component. They are quantificational. They are subject to standard scope.



## Appendix A: Full fragment

Type		Variables	Example						
truth value			true, false						
index		$i, j, k, l$	$w, x, y, z$						
entity		$d, e$	john, mary						
integers		$n, m$	1, 2						
predicate	index $\rightarrow$ proposition	$P, Q, N$	LEFT, ZEBRA						
assignment function	index $\rightarrow$ entity	$g, h$	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x</math></td> <td><math>y</math></td> </tr> <tr> <td>al</td> <td>eve</td> </tr> </table>	$x$	$y$	al	eve		
$x$	$y$								
al	eve								
information state	assign. fn. $\rightarrow$ truth value	$G, H$	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x</math></td> <td><math>y</math></td> </tr> <tr> <td>al</td> <td>eve</td> </tr> <tr> <td>ed</td> <td>ann</td> </tr> </table>	$x$	$y$	al	eve	ed	ann
$x$	$y$								
al	eve								
ed	ann								
proposition	inf. state $\rightarrow$ inf. state $\rightarrow$ truth value	$\varphi, \psi$							

$$(47) \quad G(i) \quad := \quad \{g(i) | g \in G\}$$

$$(48) \quad G|_{i=d} \quad := \quad \{g | g \in G \ \& \ g(i) = d\}$$

$$(49) \quad g[i]h \quad \Leftrightarrow \quad \text{for any index } j, \text{ if } j \neq i, \text{ then } g(j) = h(j)$$

$$(50) \quad G[i]H \quad \Leftrightarrow \quad \text{for all } g \in G, \text{ there is a } h \in H \text{ such that } g[i]h, \text{ and} \\ \text{for all } h \in H, \text{ there is a } g \in G \text{ such that } g[i]h$$

$$(51) \quad [j] \quad := \quad \lambda GH. G[j]H$$

$$(52) \quad \varphi \wedge \psi \quad := \quad \lambda GH. \exists K. \varphi(G)(K) \ \& \ \psi(K)(H)$$

$$(53) \quad \text{For any } n\text{-place predicate } P \text{ with classical logic denotation } I(P), \\ P(i_1, \dots, i_n) \quad := \quad \lambda GH. G = H \ \& \ \forall g \in G. \langle g(i_1), \dots, g(i_n) \rangle \in I(P)$$

$$(54) \quad \delta_i(\varphi) \quad := \quad \lambda GH. G(i) = H(i) \ \& \ \forall d \in G(i). \varphi(G|_{i=d})(H|_{i=d})$$

$$(55) \quad \mathbf{max}_i(\varphi) \quad := \quad \lambda GH. ([x] \wedge \varphi)(G)(H) \ \& \ \neg \exists H'. H(x) \subset H'(x) \ \& \ ([x] \wedge \varphi)(G)(H')$$

$$(56) \quad \mathbf{inside}(j) = n \quad := \quad \lambda GH. G = H \ \& \ |H(j)| = n$$

$$(57) \quad \mathbf{inside}(j/i) = n \quad := \quad \lambda GH. G = H \ \& \ \forall T \in \{H|_{i=d}(j)\}. |T| = n$$

$$(58) \quad \mathbf{outside}(j/i) > 1 \quad := \quad \lambda GH. G = H \ \& \ |\{H|_{i=d}(j)\}| > 1$$

$$(59) \quad \text{a. } \llbracket \text{students} \rrbracket = \lambda j. \text{STUDENTS}(j)$$

$$\text{b. } \llbracket \text{zebras} \rrbracket = \lambda j. \text{ZEBRAS}(j)$$

$$\text{c. } \llbracket \text{left} \rrbracket = \lambda j. \text{LEFT}(j)$$

$$\text{d. } \llbracket \text{saw} \rrbracket = \lambda i j. \text{SAW}(i)(j)$$

$$(60) \quad \llbracket \text{three}^j \rrbracket = \lambda NP. [j] \wedge N(j) \wedge P(j) \wedge \mathbf{inside}(j) = 3$$

$$(61) \quad \llbracket \text{two-two}_i^j \rrbracket = \lambda NP. [j] \wedge N(j) \wedge P(j) \wedge \mathbf{outside}(j/i) > 1 \wedge \mathbf{inside}(j/i) = 2$$

$$(62) \quad \llbracket \text{each}^i \rrbracket = \lambda NP. \mathbf{max}_i(N(i)) \wedge \delta_i(P(i))$$

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