Expressing Permission

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This paper proposes a semantics for free choice permission that explains both the non-classical behavior of modals and disjunction in sentences used to grant permission, and their classical behavior under negation. It also explains new data showing that permissions can expire when new information comes in. On the proposed approach, deontic modals update preference orderings, connectives manipulate updates rather than propositions and their logic amounts to relations between updates.

Data Using '⇒' and 'implication' to neutrally describe inferences that may be semantic or pragmatic in nature, the problem of free choice permission centers on three implications. For context, envision a perfectly informed labor representative X telling her constituents how to vote in an election. If X says (1a), her constituents can infer (1b) (Kamp 1973; von Wright 1968: 4-5).

(1) a. You may vote for Anderson or Brady
b. You may vote for Anderson and you may vote for Brady

Narrow Free Choice (NFC)
$\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}) \Rightarrow \mathsf{May}\,\mathsf{A} \wedge \mathsf{May}\,\mathsf{B}$

This implication also arises when *may* scopes under *or* (Kamp 1978: 273) — see also Zimmermann (2000), Geurts (2005), Simons (2005).

(2) a. You may vote for Anderson or you may vote Brady
b. You may vote for Anderson and you may vote for Brady

Wide Free Choice (WFC)
$\mathsf{May}\,\mathsf{A} \vee \mathsf{May}\,\mathsf{B} \Rightarrow \mathsf{May}\,\mathsf{A} \wedge \mathsf{May}\,\mathsf{B}$

Neither NFC nor WFC are valid in standard modal logic and these implications do not meet the standard cancellation test for implicatures. This makes a non-classical semantics for disjunction or modals that predicts them as entailments tempting. But that makes (1a) and (1b) equivalent, making it difficult to predict their classical behavior under negation: both are *prohibited* (Alonso-Ovalle 2006; Fox 2007).

(3) a. You may not vote for Anderson or Brady
b. You may not vote for Anderson and you may not vote for Brady

Double Prohibition (DP)
$\neg\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}) \Rightarrow \neg\mathsf{May}\,\mathsf{A} \wedge \neg\mathsf{May}\,\mathsf{B}$

Simons (2005), Barker (2010) also stress the non-implication (4) and note that in a case like *You may eat this apple or this banana* one cannot always infer that one may not eat both. Barker (2010) suggests permission is a discrete resource and logic must be sensitive to this. On this theme I add (5), which shows that a hearer can't assume permission persists after one option has been chosen.

(4) a. You may vote for Anderson or Brady
b. #You may vote for both Anderson and Brady

(5) a. You may vote for Anderson or Brady
b. You did vote for Anderson
c. #You may (still) vote for Brady

Resource Sensitivity (RS)
1. $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}) \not\Rightarrow \mathsf{May}\,(\mathsf{A} \wedge \mathsf{B})$
2. $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}) \not\Rightarrow \neg\mathsf{May}\,(\mathsf{A} \wedge \mathsf{B})$
3. $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B}), \mathsf{A} \not\Rightarrow \mathsf{May}\,\mathsf{B}$

RS1 and RS2 are non-entailments in standard modal logic, but they do emerge as implicatures on some pragmatic approaches (Barker 2010: §6.1). RS3 is neither valid in standard modal logic, nor predicted by pragmatic approaches. It is also crucial to note that if (1a)/(2a) is followed with *but I don't know which*, (1b) is no longer an implication (Kamp 1978: 271). Finally, implications like those above occur in non-permission discourse Fox (2007), a fact discussed in the full paper.

Analysis Following Kamp (1973), Kamp (1978), Lewis (1979), van Rooij (2000), $\mathsf{May}\,\phi$ is analyzed dynamically in terms of how it updates requirements/permissions π, rather than information s (a set of worlds). This dynamic analysis has two key differences. First, π distinguishes weak permission — what's consistent with what's required — and strong permission — what's been explicitly permitted; see definition (14) below and Asher & Bonevac (2005); von Wright (1968: 5). Second, there can be many competing π's and s's at play in discourse since sentences update *states* S:

(6) A **state** S is a set of **substates**: $S = \{s_1^{\pi_1}, \ldots, s_n^{\pi_n}\}$

(7) A **substate** s^π is an information state s and a practical frame π: $s^\pi := \langle s, \pi \rangle$.

Intuitively, each $s^\pi \in S$ is competing for control over the agent's actions and beliefs. Semantic clauses take the form $S[\phi] = S'$, and are read as 'S updated with ϕ is S'' Veltman (1996). Here is the basic idea of the semantics, for the simple case of $\mathsf{May}\,\mathsf{A}$, whose update effect depends on the outcome of a test:

(8) $S[\mathsf{May}\,\mathsf{A}]$: Is A is weakly permitted by all π? If *yes* do (a), if *no* do (b).

a. Add strong permission for A to each π, put each augmented π, $\mathsf{A}(\!|\pi|\!)$, in play along with π.
- Map $S = \{s_1^{\pi_1}, \ldots, s_n^{\pi_n}\}$ to $S' = \{s_1^{\pi_1}, \ldots, s_n^{\pi_n}, s_1^{\mathsf{A}(\!|\pi_1|\!)}, \ldots, s_n^{\mathsf{A}(\!|\pi_n|\!)}\}$

b. Reduce each s to $\varnothing$: $\{\varnothing^{\pi_1}, \ldots, \varnothing^{\pi_n}\}$

By putting $\mathsf{A}(\!|\pi|\!)$ in play, the speaker is allowing the hearer to choose to act on $\mathsf{A}(\!|\pi|\!)$. A successful update with $\mathsf{May\,A}$ effectively unions S with a set of substates that strongly permit A. Disjunction forms the union of updates with each disjunct, and conjunction is sequential update:

(9) 1. **Disjunction**: $S[\phi \vee \phi] = S[\phi] \cup S[\psi]$; 2. **Conjunction**: $S[\phi \wedge \psi] = (S[\phi])[\psi]$.

Together, (9.1) and (8) predict that:

(10) $\{s_1^{\pi_1}, \ldots, s_n^{\pi_n}\}[\mathsf{May\,A} \vee \mathsf{May\,B}] = \{s_1^{\pi_1}, \ldots, s_n^{\pi_n}, s_1^{\mathsf{A}(\!|\pi_1|\!)}, \ldots, s_n^{\mathsf{A}(\!|\pi_n|\!)}, s_1^{\mathsf{B}(\!|\pi_1|\!)}, \ldots, s_n^{\mathsf{B}(\!|\pi_n|\!)}\}$

With an eye to WFC, note that deontic validity is defined as follows (see also Kamp (1973); Veltman (1996); van Rooij (2000)):[1]

(11) **p-support** $S \Vdash \phi$: ϕ doesn't change any of the π's at play in S

(12) **p-consequence** $\phi_1, \ldots, \phi_n \Vdash \psi$: $\forall S \colon S[\phi_1]\cdots[\phi_n] \Vdash \psi$.

How can $S[\mathsf{May\,A} \vee \mathsf{May\,B}] \Vdash \mathsf{May\,A} \wedge \mathsf{May\,B}$ hold if each $\mathsf{A}(\!|\pi_i|\!)$ and $\mathsf{B}(\!|\pi_i|\!)$ in $S[\mathsf{May\,A} \vee \mathsf{May\,B}]$ will be further augmented to $\mathsf{B}(\!|\mathsf{A}(\!|\pi_i|\!)|\!)$ and $\mathsf{A}(\!|\mathsf{B}(\!|\pi_i|\!)|\!)$? By defining $\phi(\!|\pi|\!)$ so that $\phi(\!|\cdots(\!|\pi|\!)\cdots|\!) = \phi(\!|\pi|\!)$; see (17) below. This means that successive strong permissions are not combined, a reasonable assumption given the consistency of $\mathsf{May\,A}$ and $\mathsf{May\,\neg A}$. This much explains WFC. NFC hinges on further details.

$\mathsf{A} \vee \mathsf{B}$ creates substates: $\{s^\pi\}[\mathsf{A} \vee \mathsf{B}] = \{s_{\mathsf{A}}^\pi, s_{\mathsf{B}}^\pi\}$, where s_{A} is the A-worlds in s (atomics eliminate worlds from each s_i where they are false). So ϕ's dynamic meaning determines its alternatives in S:

(13) $alt_s(\phi) := \{a \mid a^{\pi_i} \in S[\phi]\}$

As in (Simons 2005; Aloni 2007), *may* will operate on each alternative: $\mathsf{May}\,\phi$ takes each $a \in alt_s(\phi)$ and each π, and tests whether it is consistent with what's required by π. If so, a substate featuring $a(\!|\pi|\!)$ is added to S for each $a \in alt_s(\phi)$. This predicts that $S[\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})] = S[\mathsf{May\,A} \vee \mathsf{May\,B}]$. So NFC is valid, just as WFC is. Since $\mathsf{May}\,(\mathsf{A} \vee \mathsf{B})$ will not add any substates where $\mathsf{A} \wedge \mathsf{B}$ is strongly permitted, RS1 is also explained.

Explaining DP, RS2 and RS3 depend on the way **practical frames** π are modeled:[2]

(14) $\pi := \langle R_\pi, P_\pi \rangle$ consists of **requirements** R_π and **strong permissions** P_π.

a. $R_\pi := \langle r_\pi, \sim_\pi \rangle$; read $r_\pi(w, w')$ as 'w is strictly preferable to w'', and $w \sim_\pi w'$ as 'w is just as preferable as w''. $w \nsim_\pi w'$ iff $r_\pi(w, w')$ and $w \neq w'$.

b. $P_\pi := \langle p_\pi, \approx_\pi \rangle$, interpretation parallel to R_π.

The function of π is to motivate an agent's choices. Usually, there will be a single R_π in play (the exception involves disjunctive *must*'s). Each agent decides on the best choice, given R_π, as follows:

(15) $Ch_s(R_\pi) := \{w_1 \in s \mid \nexists w_2 \in s \colon r_\pi(w_2, w_1) \;\&\; \exists w_2 \in s \colon w_1 \sim_\pi w_2\}$

This says what R_π requires: pick worlds that aren't worse than anything and are just as good as something. So a is weakly permitted by R_π just in case $Ch_s(R_\pi) \cap a \neq \varnothing$.

(16) $$S[\mathsf{May}\,\phi] = \begin{cases} S \cup \phi(\!|S|\!) & \text{if } \forall s^\pi \in S, \forall a \in alt_S(\phi) \colon Ch_s(R_\pi) \cap a \neq \varnothing \\ \{\varnothing^\pi \mid s^\pi \in S\} & \text{otherwise} \end{cases}$$

To make ϕ strongly permitted in S, $\phi(\!|S|\!)$, one makes each of $\phi's$ alternatives a strongly permitted in each π, $a(\!|\pi|\!)$. This involves overwriting P_π with $a(\!|R_\pi|\!)$, where a strict preference for each $w \in Ch_s(R_\pi) \cap a$ over each $w' \notin Ch_s(R_\pi) \cap a$ has beend added (and each of these $\langle w, w' \rangle$ removed from $\sim_\pi$):

(17) $\phi(\!|S|\!) := \{s^{a(\!|\pi|\!)} \mid s^\pi \in S \;\&\; a \in alt_s(\phi)\}$; $a(\!|\pi|\!) := \langle R_\pi, a(\!|R_\pi|\!) \rangle$; $a(\!|R_\pi|\!) := \langle a(\!|r_\pi|\!), a(\!|\sim_{r_\pi}|\!) \rangle$

[1] More precisely: $S \Vdash \phi \iff \prod_S = \prod_{S[\phi]}$, where $\prod_S := \{\pi \mid s^\pi \in S \;\&\; s \neq \varnothing\}$.

[2] Separately maintaining a strict-ordering and an indifference ordering is tedious but necessary to distinguish states that have accepted sentences that express irrational symmetric strict-preferences like $\mathsf{Must\,A} \wedge \mathsf{Must\,\neg A}$ from ones where A and $\neg\mathsf{A}$ worlds are equally good.

a. $a(\!|r_\pi|\!) := r_\pi \cup \{\langle w, w'\rangle \mid w \in Ch_s(R_\pi) \cap a \;\&\; w' \notin Ch_s(R_\pi) \cap a\}$
b. $a(\!|\sim_{r_\pi}|\!) := \{\langle w, w'\rangle \mid w \sim_{r_\pi} w' \;\&\; \text{not: } w \in Ch_s(R_\pi) \cap a \;\&\; w' \notin Ch_s(R_\pi) \cap a\}$

In a case like May A, strongly permitting A involves putting in to play a preference which makes the currently best A-worlds the best choice. This provides a picture of the preferences the hearer would have to have to do A. But, crucially, the two-part model of π captures that this preference is only presented as a *permission*. A complimentary semantics for Must ϕ is discussed in the full paper, where requirements are combined (unlike permissions) in each R_π at play in S; old R_π's don't remain.

The invalidity of RS3 comes from the fact that after updating with the information A after May (A ∨ B) will change the worlds around in s. May B changes P_π's by preferring B-worlds in s over ¬B-worlds in s. The only B-worlds that could remain are A ∧ B-worlds. If there are no such worlds, or if they are dispreferred (i.e. Must ¬(A ∧ B) has been accepted) the test imposed by May B will fail. This non-montonicity is also at play in the explanation of *but I won't tell you which* or *but I don't know which* follow ups to (1)/(2). These convey higher-order uncertainty over whether $S = \{s_1^{\pi_1}, \ldots, s_n^{\pi_n}, s_1^{\mathsf{A}(\!|\pi_1|\!)}, \ldots, s_n^{\mathsf{A}(\!|\pi_n|\!)}\}$ or $S = \{s_1^{\pi_1}, \ldots, s_n^{\pi_n}, s_1^{\mathsf{B}(\!|\pi_1|\!)}, \ldots, s_n^{\mathsf{B}(\!|\pi_n|\!)}\}$. Following Van Fraassen (1966); Stalnaker (1981), a consequence holds in such a case only if it holds on all resolutions of the uncertainty. But May A ∧ May B is p-supported by neither resolution, and neither conjunct is p-supported by both resolutions.

The crucial innovation for explaining DP is allowing negation to operate not just on information, but also on π. $\neg\phi$ removes any information ϕ would add to s, removes Per_ϕ, the permissive preferences ϕ would add to P_π, reverses them Per_ϕ^{-1}, and adds them to R_π (since Must $\neg\phi$ follows from $\neg$May ϕ).

(18) $S[\neg\phi] = \{s^{\pi \upharpoonright\!\downharpoonright \phi} - \bigcup alt_{\{s^\pi\}}(\phi) \mid s^\pi \in S\}$, where $s_i^{\pi_i} - s_j$ is read as $(s_i - s_j)^{\pi_i}$

(19) $\pi \upharpoonright\!\downharpoonright \phi := \langle R_\pi \upharpoonright \phi, P_\pi \downharpoonright \phi\rangle$;

a. $R_\pi \upharpoonright \phi := \langle (r_\pi - Req_\phi) \cup Per_\phi^{-1}, (\sim_\pi \cup Req_\phi \cup Req_\phi^{-1}) - (Per_\phi \cup Per_\phi^{-1})\rangle$
b. $P_\pi \downharpoonright \phi := \langle (p_\pi - Per_\phi), (\approx_\pi \cup Per_\phi \cup Per_\phi^{-1})\rangle$

(20) 1. $Req_\phi := \{\langle w, w'\rangle \in r_{\pi_i} \mid s^{\pi_i} \in \mathbf{0}[\phi]\}$; 2. $Per_\phi := \{\langle w, w'\rangle \in p_{\pi_i} \mid s^{\pi_i} \in \mathbf{0}[\phi]\}$

(21) 1. **Indifferent Practical Frame**: $\mathbf{I} := \langle\langle \varnothing, W^2\rangle, \langle \varnothing, \varnothing\rangle\rangle$; 2. **Initial State**: $\mathbf{0} := \{W^{\mathbf{I}}\}$;

The result of $S[\neg$May (A ∨ B)$]$ is to look at $\mathbf{0}[$May (A ∨ B)$]$ and remove any permissions it contains from S. This removes those that May A would add and those that May B would add, and converts both to requirements of their negation. So $S[\neg$May (A ∨ B)$]$ will p-support both ¬May A and ¬May B.

Comparison Unlike other semantic accounts of NFC Geurts (2005); Simons (2005); Aloni (2007), this one clearly predicts DP as an entailment. (Aloni 2007: 80) can predict it with a particular selection of A ∨ B's alternatives, but offers no rationale for this selection. The tradeoffs here are complex and will be discussed in the full paper. But, the approach here shows that a less stipulative semantic explanation of DP is possible. (Barker 2010: §5) treats DP as an implicature, based on uncooperative or uninformed speakers blocking the implication. As proposed above, cases with this feature can be treated as higher order (hearer or speaker) uncertainty about S.

(Aloni 2007: 91)'s semantic analysis does not capture WFC. The existing semantic options are across-the-board LF movement (Simons 2005), thereby reducing WFC to NFC, and treating *or* as systematically ambiguous (Barker 2010: 25). Both approaches face over-generation issues. LF movement is a type-driven process, which makes it hard and *ad hoc* to limit it to particular modals and connectives of the same type. Yet, *You may vote for Anderson and you may vote for Brady ≠ You may vote for Anderson and Brady.* (Barker 2010: 25) grants that similar complexities apply to restricting the distribution of ambiguous connectives and does not attempt to navigate them. Pragmatic accounts like Zimmermann (2000) and semantic accounts like Geurts (2005) that do capture WFC require modifying the semantics in a way makes DP difficult to predict. As (van Rooij 2010: 24) notes, it is difficult to see how a pragmatic approach to WFC can succeed with a classical semantics for disjunction. The challenge is then to capture WFC alongside DP. Since the analysis offered here predicts WFC and DP to be valid without movement or an ambiguity in *or*, it provides at least an interesting alternative solution to these extremely difficult challenges.

Barker (2010) predicts only RS1 and RS2, not RS3, similarly for Simons (2005); Aloni (2007). I am not aware of a pragmatic account that captures RS3, and some struggle with RS1 and RS2 — though Schulz (2007) captures RS1 and RS2.

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